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Development of an Improved Altitude Estimation Technique for a Minimum Configuration Multilateration System

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ABSTRACT— Multilateration (MLAT) system is known to have high altitude estimation error limiting its application in 3-dimensional (3-D) aircraft surveillance. This paper proposed a technique based on vector polynomial addition of the second-order time difference of arrival (TDOA) quadratic equations aimed at reducing thealtitude estimation error of a 3-D minimum configuration MLAT system. The proposed technique is validated at some randomly selected aircraft positions at different flight level (FL)s by comparing with the conventional technique. Monte Carlo simulation result shows a reduction in the altitude root mean square error (RMSE) by at least 50% using the proposed technique based on a square ground sensor (GS) configuration. Furthermore, the proposed technique enables for the implementation of the minimum configuration MLAT system in a 3-D scenario having an altitude RMSE in compliance with the reduced vertical separation minimum (RVSM) initiative.

KEYWORDS—multilateration, TDOA, altitude estimation, polynomial addition, RVSM

INTRODUCTION

Multilateration (MLAT) system is a type of wireless positioning system used by the air navigation service provider (ANSP) for surveillance purposes [1]. The aircraft position estimation (PE) process of the system is in two stages [2, 3]: (1) time difference of arrival (TDOA) estimation and; (2) localization with a lateration algorithm. The TDOAs are estimated from the aircraft transponder emission detected at spatially located ground sensor (GS) pairs [1, 4, 5]. The TDOA estimates are used with the lateration algorithm to estimate the aircraft position which is displayed on the screen at the air traffic control (ATC) center [1].

Aircraft altitude estimation above 29,000 ft. is critical as the Vertical Separation Minima (VSM) between two consecutive flight level (FL)s is reduced from 2,000 ft. to 1,000 ft. [6]. This is one of the initiatives by the International Civil Aviation Organization (ICAO) to increase airspace capacity to meet up with current and future global air traffic demands [7]. MLAT systems are known to have high aircraft altitude estimation error [8, 9]. With a 300 ft. maximum allowed system altitude report error for all FLs set by the International Civil Aviation Organization (ICAO) in accordance with the Reduced VSM (RVSM) initiative [6], it is difficult to implement the system for 3-D surveillance purposes. Several techniques have been proposed to able the implementation of MLAT system in 3-D scenario [8, 10, 11]. It was suggested in [8] that altitude information of an aircraft should be obtained from other sources such as Mode C of secondary surveillance radar (SSR) interrogator reply and then incorporated in the MLAT system PE process. The Mode C altitude information has been shown to be an actual measure of pressure and is not always accurate as it varies with time and place [12]. In [13, 14], a 3-D minimum configuration MLAT system closed-form lateration algorithm is developed. The MLAT system consist of four spatially located GSs each connected to a central processing station (CPS) [1]. This paper presents a technique to improve the altitude estimation accuracy of the lateration algorithm. Monte Carlo (MC) simulation is used to determine the improvement in the altitude estimation process at some randomly selected aircraft positions.

The rest of the paper is organized as follows. Section II presents the mathematical derivation of the MLAT system PE methodology and the conventional technique for altitude estimation. The proposed altitude estimation technique is presented in section IIIwhich is followed by simulation result and discussion in section IV Finally, the conclusion is presented in section V.

MLAT SYSTEM POSITION ESTIMATION METHODOLOGY

This section first gives a summary of the closed-form approach to the development of the 3-D minimum configuration reference pair lateration algorithm as presented in [14]. This is followed by the conventional technique to the estimation of altitude based on the minimum configuration closed-form lateration algorithm.

Closed-Form Reference Pair Lateration Algorithm Development

Let $\mathbf{x}_e = (x, y, z)$ be the coordinate of an aircraft in the 3-D Euclidean space while $\mathbf{s}_i = (x_i, y_i, z_i)$, $\mathbf{s}_j = (x_j, y_j, z_j)$, $\mathbf{s}_k = (x_k, y_k, z_k)$ and $\mathbf{s}_l = (x_l, y_l, z_l)$ respectively be the coordinate of the *i*-th, *j*-th, *k*-th and *l*-th ground sensors. Using the *i*-th and *j*-th GS are reference for TDOA estimation, four TDOA hyperbolic equations are obtained as follows [14]:

$$\tau_{ik} = \frac{1}{c}\left(\left\|\mathbf{x}-\mathbf{s}_i\right\| - \left\|\mathbf{x}-\mathbf{s}_k\right\|\right) \quad (1)$$

$$\tau_{il} = \frac{1}{c}\left(\left\|\mathbf{x}-\mathbf{s}_i\right\| - \left\|\mathbf{x}-\mathbf{s}_l\right\|\right) \quad (2)$$

$$\tau_{jk} = \frac{1}{c}\left(\left\|\mathbf{x}-\mathbf{s}_j\right\| - \left\|\mathbf{x}-\mathbf{s}_k\right\|\right) \quad (3)$$

$$\tau_{jl} = \frac{1}{c}\left(\left\|\mathbf{x}-\mathbf{s}_j\right\| - \left\|\mathbf{x}-\mathbf{s}_l\right\|\right) \quad (4)$$

Where: c = 3×10^8 m/s, $\hat{\tau}_{ik}$ and $\hat{\tau}_{il}$ are respectively the TDOA measurements obtained using the *i*-th reference GS with the *k*-th and *l*-th non-reference GS while $\hat{\tau}_{jk}$and$\hat{\tau}_{jl}$ are respectively the TDOA measurements obtained using the *j*-th reference GS with the k-th and l-th non-reference GS. The symbol $\| \;\|$ denotes the 2-norm operator.

In practical application, the TDOA measurements in (1) to (4) are obtained with error. By modelling the TDOA error as zero mean Gaussian random variable with a normal probability density function [15], the estimated TDOAs in (1) to (4) respectively are:

$$\hat{\tau}_{ik} = \tau_{ik} + N(0,\sigma) \quad (5)$$

$$\hat{\tau}_{il} = \tau_{il} + N(0,\sigma) \quad (6)$$

$$\hat{\tau}_{jk} = \tau_{jk} + N(0,\sigma) \quad (7)$$

$$\hat{\tau}_{jl} = \tau_{jl} + N(0,\sigma) \quad (8)$$

Where:σ is the TDOA error standard deviation (SD) with typical values ranges from 0 nsec to 20 nsec [1, 16]. The TDOA error is assumed to include error due to noise in the signal, GS clock synchronization error, and quantization error.

Algebraically manipulatingEq. (1) to Eq. (4) as previously done in [13, 14] results in a pair of 3-D plane equation in the form [14]:

$$A_{i,k,l} = xB_{i,k,l} + yC_{i,k,l} + zD_{i,k,l} \quad (9)$$

$$A_{j,k,l} = xB_{j,k,l} + yC_{j,k,l} + zD_{j,k,l} \quad (10)$$

Where: the coefficients of Eq. (9) and Eq. (10) are functions of the TDOA measurements and GS coordinate which can be found in [14]. The detailed derivation of Eq. (9) and Eq. (10) from Eq. (1) to Eq. (4) is not within the scope of this work but can be found in [13, 14].

Conventional Technique to Altitude Estimation

The conventional technique to obtain the altitude of the aircraft using Eq. (9) and Eq. (10) as previously done in [13, 14] involves first expressing the horizontal coordinates of the aircraft (x, y) as a function of its altitude (z) as shown in Eq. (11) and Eq. (12).

$$x = K - zI \quad (11)$$

$$y = I - zJ \quad (12)$$

This is followed by substituting Eq. (11) and Eq. (12) into Eq.(1) to obtain a second-order quadratic equation as follows [13]:

$$Q_{ik} z^2 + 2P_{ik} z + Q_{ik} = 0 \quad (13)$$

Where: the coefficients of Eq. (11) to Eq. (13) can be found in [13].
Lastly, the solution to z in (13) is obtained as:

$$z = \frac{-P_{ik}}{2O_{ik}} \pm \frac{\sqrt{P_{ik}^2 - 4O_{ik}P_{ik}}}{2O_{ik}} \quad (14)$$

Since the altitude of the aircraft cannot be a negative value, the estimated altitude is:

$$\hat{z} = \frac{-P_{ik}}{2O_{ik}} + \frac{\sqrt{P_{ik}^2 - 4O_{ik}P_{ik}}}{2O_{ik}} \quad (15)$$

Eq. (15) is the conventional technique to estimating the altitude of the aircraft for the minimum configuration MLAT system lateration algorithm.

Proposed Altitude Estimation Technique Development

In this section, the technique to improve the altitude estimation of the minimum configuration multilateration system is presented. The use of a single TDOA measurement to solve for the aircraft altitude is not optimum. An improvement in the altitude estimation can be achieved by using all the four TDOA measurements that is using Eq. (2) to Eq. (4) in addition to Eq. (1). The remainder of the second-order quadratic equations obtained using the TDOA measurements in Eq. (2) to Eq. (4) respectively are:

$$Q_{il} z^2 + 2P_{il} z + Q_{il} = 0 \quad (16)$$

$$Q_{jk} z^2 + 2P_{jk} z + Q_{jk} = 0 \quad (17)$$

$$Q_{jl} z^2 + 2P_{jl} z + Q_{jl} = 0 \quad (18)$$

Vector polynomial addition of Eq. (13), Eq. (16) to Eq. (18) results in another second-order quadratic equation expressed as follows:

$$O_{ijkl} z^2 + 2P_{ijkl} z + Q_{ijkl} = 0 \quad (19)$$

Where: the coefficients of (19) are:

$$O_{ijkl} = O_{ik} + O_{il} + O_{jk} + O_{jl} \quad (20a)$$

$$P_{ijkl} = P_{ik} + P_{il} + P_{jk} + P_{jl} \quad (20b)$$

$$Q_{ijkl} = Q_{ik} + Q_{il} + Q_{jk} + Q_{jl} \quad (20c)$$

Thus, the estimated altitude of the aircraft by solving for z using Eq. (19) is:

$$\hat{z} = \frac{-P_{ijkl}}{2O_{ijkl}} + \frac{\sqrt{P_{ijkl}^2 - 4O_{ijkl}P_{ijkl}}}{2O_{ijkl}} \quad (21)$$

Eq. (21) is the proposed altitude estimation technique and will result in less altitude error due to the averaging effect as shown in Eq. (15).

Simulation Result and Discussion

In this section of the paper, the improvement in the altitude estimation using the proposed technique based on Eq. (21) is determined by comparing with the conventional technique based on Eq. (15) as previously done in [13, 14] is presented. The altitude root mean square error (RMSE) is used as the performance measure for comparison and is mathematically expressed as follows:

$$Alt_{rmse} = \sqrt{\frac{1}{500}\sum_{n=1}^{500}\left(\hat{z}_n - z\right)^2} \qquad (22)$$

Where: z is the known altitude of the aircraft and $\hat{z}_n$ is the estimated altitude at the n-th MC simulation realization.

For the analysis, aircraft at six randomly selected positions are considered each at a different FLs as shown in Table 1.

TABLE I: POSITION OF AIRCRAFT FOR ALTITUDE ESTIMATION IMPROVEMENT

Aircraft location	**Coordinates**		
	x ***(km)***	***y*** ***(km)***	***z*** ***(ft.)***
A	50	80	29,000
B			31,000
C	80	50	33,000
D			35,000
E	100	30	37,000
F			39,000

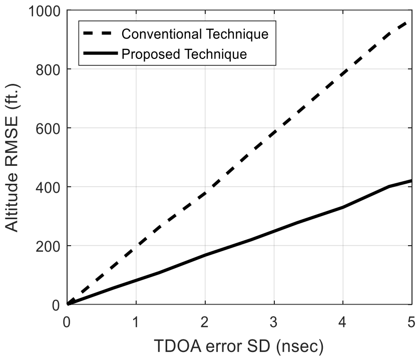
By varying the TDOA error SD from 0 nsec to 5 nsec, the relationship between the TDOA error SD and the altitude RMSE at each of the selected aircraft positions with coordinate defined in Table I is determined for both the conventional and proposed technique. Fig. 1 shows the relation between the TDOA error SD and altitude RMSE at the selected aircraft positions defined in Table I. Irrespective of the aircraft position, the altitude RMSE increases with the TDOA error SD from 0 nsec to 5 nsec. At a fixed TDOA error SD, the altitude RMSE varies with aircraft position. Comparison between the two techniques at each of the selected aircraft positions shows an improvement in the altitude estimation accuracy by the proposed technique through the reduction in the altitude RMSE. Table II shows the altitude RMSE comparison between the conventional and proposed technique at different aircraft positions using a receiver with TDOA error SD of 1 nsec [16]. At aircraft position A, the altitude RMSE obtained using the conventional and proposed techniques are 196 ft. and 82 ft. respectively. The absolute difference in the altitude RMSE is about 114 ft. which is about 58% of the error obtained using the conventional technique. This means that, the proposed technique reduced the altitude RMSE of the aircraft at position A by about 58% compared to the conventional technique.

TABLE II. ALTITUDE RMSE COMPARISON AT TDOA ERROR SD OF 1 NSEC

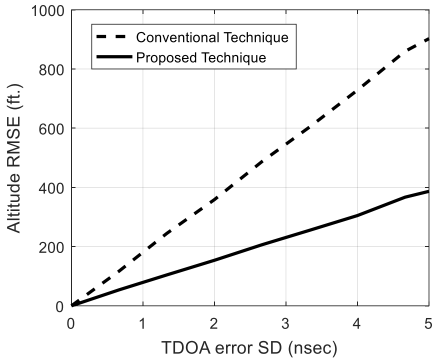
Aircraft location	**Altitude RMSE (ft.)**		**Altitude RMSE Reduction (ft.)**
	Conventional technique	***Proposed technique***	
A	196	82	114
B	181	79	102
C	219	75	144
D	199	71	128
E	996	264	732
F	893	250	643

Extending the analysis to the remaining aircraft positions, the absolute altitude RMSE differences at aircraft positions B, C, D, E, and F are 102 ft., 144 ft., 128 ft., 732 ft, and 643 ft. respectively. The percentage reduction in the altitude RMSE obtained using the proposed technique compared to the conventional technique at these aircraft positions are ~56%, ~65%, ~64%, ~73% and ~72% respectively. On the average, based on the selected aircraft position, there is about 60% reduction in the altitude RMSE using the proposed technique based on Eq. (21) compared to using the conventional technique based on Eq. (15).

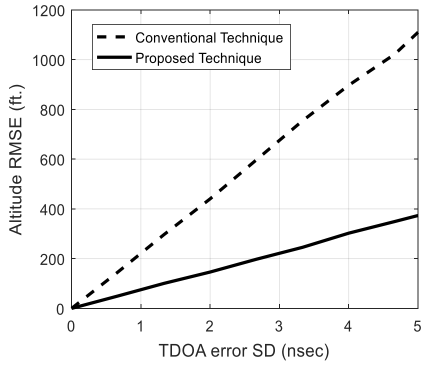
As earlier mentioned, the maximum allowed altitude RMSE for compliance with the ICAO RVSM at all FLs in 300 ft. At aircraft positions E and F, the altitude RMSE values using the conventional technique respectively are 996 ft. and 893 ft. which are 696 ft. and 593 ft. above the approved ICAO



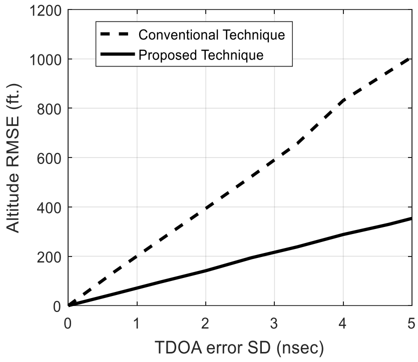
(a) Aircraft at position A



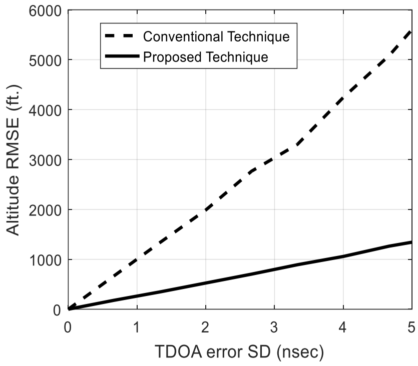
(b) Aircraft at position B



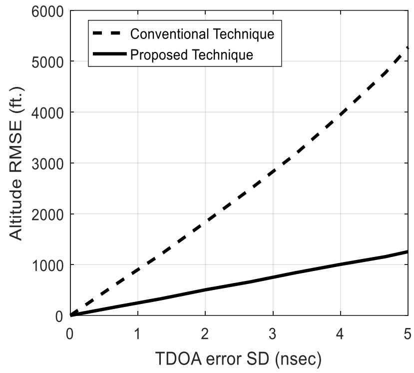
(c) Aircraft at position C



(d) Aircraft at position D



(e) Aircraft at position E



(f) Aircraft at position F

Fig. 1. Altitude RMSE versus TDOA error SD comparison at some selected aircraft positions.

RVSM standard thus, not acceptable. Using the proposed technique, the altitude RMSE at these selected aircraft positions are 264 ft. and 250 ft., which are below the 300 ft. maximum, approve standard. This means that the proposed technique reduced the altitude RMSE at these selected aircraft positions to be within approved standard set for the ICAO RVSM initiative making it possible for implementation in 3-D aircraft position estimation scenarios.

CONCLUSION

In this paper, a technique to improve the altitude estimation accuracy of a 3-D minimum configuration MLAT system is suggested. The proposed technique is validated by comparing it with the conventional technique at some selected aircraft position using MC simulation. Simulation result shows the proposed technique improved the altitude estimation accuracy through a reduction in the altitude RMSE by at least 50% compared to the conventional technique. Compliance verification with the ICAO RSVM standard for 3-D implementation shows that the altitude RMSE obtained using the proposed technique is less than the 300 ft. maximum error. With the use of the proposed technique, MLAT system can be implemented in a 3-D scenario and the altitude RMSE obtained will be within the proved set standard.

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